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Preparing Teachers to Lead Mathematics Discussions

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Background/Context: *Discussion is central to mathematics teaching and learning, as well as to mathematics as an academic discipline. Studies have shown that facilitating discussions is complex work that is not easily done or learned. To make such complex aspects of the work of teaching learnable by beginners, recent research has focused on approaches to teacher education that decompose practice into smaller tasks or routines that can be articulated, unpacked, studied, and rehearsed.*

Purpose/Objective/Research Question/Focus of Study: *Drawing on the elements of Grossman et al.'s (2009) framework for pedagogies of practice in professional education, this article describes an approach to analyzing teaching practice that supports the design of teacher education in which novices to learn how to engage in high-leverage teaching practices and simultaneously develop a sense of why the work is done.*

Setting: *The context for the study is a mathematics methods course for prospective elementary teachers.*

Research Design: *The study involved qualitative analysis of both the design of the course and its implementation.*

Conclusions/Recommendations: *Results from this study suggest that: (1) Decomposing teaching into nested practices of varying grain sizes that maintain the connection between techniques and domains can support beginning teachers in attending simultaneously to the how and the why of practice, as well as provide a map of teaching that teacher educators can use to support the learning of teaching. (2) Nesting early approximations of practice inside subsequent ones is a way to support beginning teachers in building toward recomposed teaching practice. As practices are nested, teacher educators can increase the complexity of the approximations by adjusting the content and authenticity of the context of practice as well as by decreasing the amount scaffolding provided. (3) Assessment is vital to pedagogies of practice. Decompositions, approximations, and representations of practice need to be developed in ways that support assessment for the array of teacher education purposes. (4) Because subject matter is at the heart of teaching, it is crucial to attend to the ways in which the content interacts with pedagogies of practice.*

Discussions are a central component of mathematics instruction. In the context of class discussions, teachers lead students to examine multiple solutions to mathematics problems; compare alternative representations, terms, and solutions; and make connections. Successful discussions require substantial teaching skill. This is because students must be helped to engage in complex mathematical practices such as giving explanations, making connections, and using representations, and, at the same time, teachers' moves must be contingent on what students say and do. Furthermore, leading a discussion requires mathematical knowledge for teaching (MKT; Ball, Hill, & Bass, 2005), given that teachers need to size up mathematical ideas flexibly, frame strategic questions, and keep an eye on core mathematical points (Sleep, 2009).

This article focuses on an approach to helping beginning teachers learn to lead whole-class discussions. This approach is grounded in naming and describing practices that are central to the work of mathematics teaching. The articulation of practice is essential for the design of learning experiences, for assessing teaching (Moss, Boerst, & Duckor, 2008), for reflecting on and improving teaching (Grossman & McDonald, 2008), and for enhancing dialogue among professionals (Shulman, 1999). We draw on the work of Grossman and colleagues (2009), who defined "pedagogies of practice in professional education" as comprising three key elements: (1) *representations of practice* (i.e., the ways in which practice is represented to learners and what those representations make visible); (2) *decompositions of practice* (i.e., the ways in which practice is "broken down into its constituent parts for the purposes of teaching and learning" [p. 2]); and (3) *approximations of practice* (i.e., the opportunities for beginning teachers to engage in the practice in ways that approach its enactment in the profession). In this article, we use this framework to describe and analyze our efforts to develop beginning teachers' skills in leading a whole-class discussion in mathematics.

The article begins with an overview of the importance of discussions in mathematics teaching and why beginning teachers need support in learning to implement them well. We then describe the context for our work on teaching beginning teachers to lead discussions. The majority of the article focuses on describing our approach to decomposing teaching practice and on analyzing how we have used these to design learning opportunities for beginning elementary teachers. We conclude with a discussion of the implications of our work for conceptualizing pedagogies of practice in professional education.

DISCUSSIONS IN MATHEMATICS TEACHING

Over an extended period, there have been advocates for increasing the amount and quality of discussion in mathematics classrooms (e.g., Chapin, O'Connor, & Anderson, 2003; Hoyles, 1985; National Council of Teachers of Mathematics [NCTM], 1991; Lampert & Cobb, 2003; Smith, Hughes, Engle, & Stein, 2009). This attention has not yielded a consensus on what parameters define a mathematics discussion. Pimm (1987) argued that although the term *discussion* provides an opportunity to specify a "category of verbal interchange," it is typically not well described or bounded. In some ways this is understandable, given that mathematics discussions are shaped by a range of purposes (e.g., to initiate or extend learning, investigate conjectures, share solutions and strategies, unpack complex ideas), content (e.g., concepts, procedures, practices, tasks, problems, errors, and inventions within and across many strands of disciplinary content), and contexts in which discussions occur (e.g., the size of the group, who "leads" the discussion). It follows that discussions are not monolithic, but come in a variety of forms as a function of both design and necessity. Nevertheless, it is possible to identify useful characterizations of discussions. The type of discussion that we focus on in this article is similar to the form described by Chapin et al.:

> In a whole-class discussion the teacher is in charge of the class. . . . However, the teacher is not primarily engaged in delivering information or quizzing. Rather, he or she is attempting to get students to share their thinking, explain the steps in their reasoning, and build on one another's contributions. The whole-class discussions give students a chance to engage in sustained reasoning. The teacher facilitates and guides quite actively, but does not focus on providing answers directly. Instead the focus is on the students' thinking. (p. 17)

Several elements of this description bear elaboration. First, for a mathematics discussion to provide the opportunity for participants to engage in sustained reasoning, it must have a focus. This focus may be a concept, procedure, problem, or recently completed activity that can evolve through interaction. This focus has sufficient depth to support dialogue for an extended period. Second, discussions require participants to make contributions and also to take up what is shared by others. Participants share responsibility for accomplishing mathematical work in the context of collective talk. They assume this responsibility by speaking and listening, and often though the use of representations and tools. Third, the teacher is an active participant in a discussion and not a passive observer. The teacher contributes ideas and encourages students to participate, attend to the contributions of others, and engage with the mathematical focus of the discussion. In sum, the whole-class mathematics discussion that is the focus of this article occurs when classroom participants, including the teacher, engage in focused and collective dialogue about mathematics.

The importance of discussions in mathematics teaching is rooted in the nature of mathematics as a discipline, the ways in which students learn, and the context of teaching. Mathematical knowledge is established through interaction in a community of practice. It requires articulation of ideas, extensive and often critical public interaction, and building on ideas that are shared and established in the community (Ball & Bass, 2000). Discussions are also the primary mechanism for students' learning of mathematics, in particular, for promoting conceptual understanding (Michaels, O'Connor, & Resnick, 2008). At the same time, discussions are occasions for teachers to learn what students think, thereby providing a useful source of information for instructional decision making (Pimm, 1987). Discussions provide opportunities for students to build on prior knowledge, connect ideas, and learn to see things from other perspectives—all foundational to learning (Confrey, 1990; NCTM, 2000). They offer a means of accommodating diverse ideas and equitably providing all students with explicit access to mathematical ideas (Ball, Hoover, Lewis, Bass, & Wall, 2003; NCTM, 1991). Discussions also can support students in learning disciplinary norms and conventions (Rittenhouse, 1998; Yackel & Cobb, 1996).

Furthermore, discussions have become a central instructional method in a growing number of curriculum materials (e.g., TERC, 1998).

Although discussions are central in mathematics teaching and learning, they are challenging to carry out effectively for a number of reasons. Smith and colleagues (2009) noted the difficulty of maintaining the cognitive demand, handling unanticipated ideas, and honoring student contributions. Lampert (2001) described the tension in managing and supporting the learning of the whole class while simultaneously attending to individual student contributions. Teachers draw on substantial knowledge and skill to maintain collective engagement in discussions in order to avoid more simplistic and less effective “show-and-tell” sessions (Ball, 2001) and long, relatively aimless strings of dialogue. The mathematical knowledge demands are substantial as teachers simultaneously use mathematical language, interpret student thinking, and represent mathematical ideas. These demands are compounded by the relatively brief amount of time that a mathematics teacher has between hearing a student contribution and the need to make a move that keeps the unfolding interaction in motion and headed in a productive direction. For beginning teachers, this type of work is particularly difficult (Heaton, 2000; Sherin, 2002) Because discussions are essential to mathematics instruction but difficult to implement effectively, it is particularly important that beginning teachers have structured guidance, experience, and feedback in preparation to teach mathematics.

DATA AND METHODS

In this section, we describe the context for our work on teaching beginning teachers to lead discussions: a mathematics methods course for beginning elementary teachers. We provide an overview of the course and then describe our approach to decomposing practice.

CONTEXT FOR OUR WORK

The course that serves as the context for this article is an elementary mathematics methods course for beginning teachers. For over a decade, a rotating group of instructors has collaborated to teach the multiple sections of the course that are offered each year.[1] This group of instructors, now known as the Mathematics Methods Planning Group (MMPG), meets weekly to collectively design, plan, and implement the course. Over the years, MMPG has assembled an extensive library of course materials, including detailed lesson plans, presentation slides, assignments, assessments, and grading tools. The prior year’s materials are used as the starting point for the current year; each year, the materials are further developed and improved. The data analyzed in this article are from the 2006 version of the course. We chose this year to analyze because we documented that year especially extensively.[2] We video-recorded all of one section’s sessions and collected beginning teacher work across all sections. However, the ideas and activities described in this article built on work of several prior years and are still used in our current courses.

In 2006, there were four sections of the course. Two were offered as part of our undergraduate teacher education program and two as part of our postbaccalaureate (master’s) teacher education program. While they were taking our methods course, beginning teachers in both programs had a field placement in an elementary classroom, where they worked with the cooperating classroom teacher and a university-based field instructor. Each section of the course met for 3 hours each week and comprised 13 class sessions plus a final exam. Beginning teachers completed weekly assignments and a number of field-based projects, including conducting a student interview and teaching a whole-class mathematics lesson from the textbook used in their field-placement classroom.

OUR APPROACH

The underlying model for our methods course centers on providing opportunities for beginning teachers to learn in, from, and for practice (Lampert, 2010). We seek to help beginning teachers develop their skills at *doing* teaching, not simply analyzing it. We focus on teaching practices that are crucial for pupils’ learning and that beginning teachers can learn to perform proficiently. We call these “high-leverage practices” (Ball, Sleep, Boerst, & Bass, 2009).

Designing a course focused on high-leverage practices is challenging for a number of reasons inherent to the work of teaching. First, teaching is complex and not well articulated (Morris & Hiebert, 2009), making it difficult to specify the content to be taught. Second, teaching is hard to "see"—both because some of the work occurs in teachers' heads and because practices that are visible go undetected (Lewis, 2007). And third, teaching is interactive, which means that much of practice is contingent on the actions and responses of the particular students being taught. All these factors make teaching both difficult to teach and hard to learn to do well.

To make the complex work of teaching mathematics more learnable by beginners, we aim to decompose practice into smaller tasks or routines that can be articulated, unpacked, studied, and rehearsed (Grossman et al., 2009; Grossman & Shahan, 2005; Lampert, 2001, 2005). Such decomposition temporarily reduces complexity by holding some aspects of teaching "still" or by routinizing some components of the work so that beginning teachers can attend to and practice particular skills or focus on specific problems. A foundational challenge in decomposing practice is to navigate a satisfying answer to the question, *What is the work that mathematics teachers do?* The response needs to reflect the actual work of mathematics teaching, be both comprehensible by and satisfying to beginning teachers, and be conducive to instruction in teacher education.

To identify the high-leverage practices on which to focus in our course, MMPG began to work systematically to unpack and articulate the work of mathematics teaching. Building on past work and the diverse knowledge and experiences of individual group members, MMPG engaged in an iterative process of examining records of elementary mathematics teaching; consulting standards, professional literature, and research on teaching; and investigating teacher preparation in other subjects. These analyses yielded lists and descriptions of candidate practices along with criteria for judging the comparative "leverage" that the practices provide in mathematics teaching, in teacher education, and for beginning teachers (see Ball et al., 2009, for a list of criteria).

In recent years, the work of MMPG has merged with and benefitted from other projects at the University of Michigan, in particular, the NSF-funded Developing an Integrated Assessment System (DIAS) Project (Moss et al., 2008), which has developed detailed descriptions of high-leverage practices for mathematics teaching and an infrastructure by which these practices can be meaningfully assessed, and the Teacher Education Initiative (TEI; 2009), which is leading the redesign of the School of Education's practice-focused teacher education program. Our current conceptualization of the work of leading a discussion in mathematics has been informed by the work of these groups.

In the next section, we describe the conceptualization of teaching that has resulted from our efforts to decompose practice in order to teach teaching to beginning teachers.

CONCEPTUALIZING TEACHING AS NESTED PRACTICES OF VARYING GRAIN SIZES

Grossman and her colleagues (2009) argued for a pedagogy of practice in professional preparation that begins with "decomposition," or breaking practice into its constituent parts. Traditionally, teacher education has decomposed practice in two main ways. One approach has been to focus on large domains of teaching practice, and the other has been to work on particular techniques of teaching. These two approaches differ with respect to grain size of practice. We discuss each approach in this section and then propose an alternative approach that has formed the basis of our work on the decomposition of practice.

FOCUSING ON LARGE DOMAINS OF PRACTICE

Parsing teaching into large domains of practice (e.g., creating a classroom community, assessing, planning) is common in professional standards and resources (Hiebert et al., 1997; NCTM, 1991; National Board for Professional Teaching Standards, 2001) as well as in teacher preparation (Bahr & Garcia, 2010; Interstate Teacher Assessment and Support Consortium, 1992; Perrone, 2000), and there are several advantages to this approach. For example, it is possible to "cover" teaching practice with a relatively small set of domains that can be easily named. These major domains can be seen in even dramatically different approaches to teaching. As a result, beginning teachers perceive these domains as relevant to their professional learning. Furthermore, because professional literature and standards are organized at this grain size, beginning teachers simultaneously learn the infrastructure and language of the profession. Decomposing teaching into large domains also has affordances for teacher educators. A focus on domains yields a manageable inventory of practices that can be mapped onto typical teacher-education time frames (e.g., a semester). Domains are substantive and thus help portray teaching as complex work that requires knowledge and judgment. As a result, teacher education does not seem reductionist because teaching is not presented as disconnected or as something that can be boiled down to simple acts.

Although decomposing practice into domains is useful, it also poses challenges when used as the basis for professional preparation. One problem is that the scale and complexity of domains make it challenging to scaffold beginning teachers' engagement in the work. Consider, for example, trying to support beginning teachers' work on *learning community*. This is a complex territory of work, and knowing that it is important is insufficient to learning specific things to *do* to establish and maintain a learning community in a classroom. A focus on domains often leads to learning experiences that help develop beginning teachers' commitments but that offer little pedagogical traction. In trying to support beginning teachers' learning to teach, such large-grain decompositions seemed to us insufficient as a framework for teaching practice.

FOCUSING ON PARTICULAR TECHNIQUES

A second common approach to decomposition focuses on teaching beginning teachers techniques or procedures, such as using the overhead, taking anecdotal notes, and managing the use of manipulatives. This approach counters the frequent criticism of teacher education as too "theoretical" and not focused on practice. A focus on techniques is common in "how to" resources available at teacher stores, on the Internet, in teacher workshops, and in local conference presentations. "Practical" guidance has historical roots in teacher training models of professional preparation (e.g., microteaching; MacLeod, 1987). There are several advantages to decomposing practice at a finer grain size. A focus on techniques emphasizes the specifics of the work that teachers do. Processes can be articulated, giving beginning teachers tools they can implement in the classroom. Techniques can be tried out repeatedly, which provides beginning teachers opportunities to refine their practice and to learn about contextual influences. Teachers who mentor beginning teachers often appreciate the need for beginners to learn techniques and are able to readily provide contexts in which they can be tried.

However, decomposing practice into techniques also has limitations. Although beginning teachers do develop skills, they are likely to emerge from professional preparation with a relatively small tool kit. This can cause anxiety because they know from their own experience in schooling, from observations of teachers, and from their own trials of techniques that there is much more to teaching than can be managed by any set of techniques. Emphasizing technique without adequate framing can also make it difficult to keep an eye on purposes. An orderly system for allocating turns in class may make a classroom appear well managed, but if that system impedes pupils' substantive discussion in class, then it is empty technique (Palincsar, personal communication, January 2010). Focusing on techniques can also create challenges for teacher educators. A collection of techniques may represent teaching as a set of simple actions that do not require substantive knowledge or judgment. Moreover, the scale and scope of decomposition can become unwieldy. For example, some teaching techniques focus on supporting different work formats (e.g., whole class, cooperative group, individual); others are associated with the mechanics of a particular instructional activity (e.g., guided reading, science investigations). The lists are long and not simple to organize into coherent schemes for sensible learning and use. Furthermore, not all techniques are used in all contexts (e.g., homework checking, spelling reviews, taking roll); thus, it can be challenging for teacher educators to make durable connections to the field.

Although there are strengths and weakness to both approaches to decomposing practice, neither the large-domain approach nor the technique approach suffices as a framework for teacher education. This can be seen both from the standpoint of beginning teachers who need to learn *how* to engage in the work and develop a sense of *why* the work is done, and from the standpoint of teacher educators who are responsible for preparing teachers to do the work of teaching.

Our approach to decomposing practice is grounded in an alternative perspective: We decompose teaching into nested practices of varying grain sizes that connect techniques and domains. This way of conceptualizing decomposition informs our articulation of the content of our course, as well as our choice of representations of practice and the design and sequencing of approximations of practice. We elaborate it in the next section.

DECOMPOSITION AS A PROCESS OF "GETTING DOWN TO SPECIFICS" AND "RISING ABOVE THE DETAILS"

Our approach to decomposing teaching into nested practices of varying grain sizes can, in some ways, be seen as a combination of the two approaches described earlier. In other words, we start at the domain level and continually specify how that practice can be implemented until we get down to techniques that can be specified and taught to and worked on by beginning teachers. However, a key trait of our approach is that, along the way, we maintain the connection between domains and techniques through the articulation of intermediate practices.

To decompose practice in such a way, we begin by specifying the constituents of specific domains of mathematics teaching. For example, specifying the leading of a discussion includes brief descriptions of:

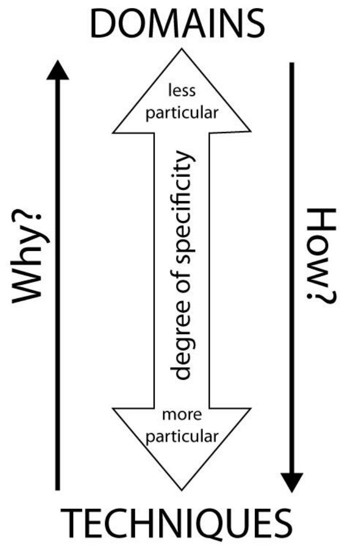
- the nature of the work (e.g., inviting and coordinating participation from students, making contributions, representing and recording ideas);
- the mathematics that may be discussed (including, but not limited to, concepts, procedures, practices, tasks, problems, questions, conjectures, errors, and inventions); and
- the nature of the interaction that happens in a discussion (noting modes of communication and distributed engagement; DIAS Project, 2009).

We then articulate strings of smaller and increasingly specified teaching practices that are nested within this domain until we reach the level of technique. For example, through successive parsing the work of leading a discussion, we reach the technique of “revoicing,” described by O’Connor and Michaels (1993) as restating a contribution though the use of repetition, rephrasing, or expansion to be generative. Revoicing can be used for multiple purposes (e.g., to emphasize a key point or question), reflects orientations toward teaching and learning that we are trying help beginning teachers develop (e.g., trying to listen to, respect, and use what children say), and can be rehearsed and easily deployed during engagement with children. It is also grounded in professional literature on which we can draw.

In addition to the domains and techniques that are articulated, this approach generates two other interrelated products that are of use in the design of teacher education. One product is the identification of *intermediate-level practices*. In the case of leading a discussion, successive decomposition yields intermediate practices such as following up on student responses and clarifying student thinking before reaching techniques like revoicing. Each of these intermediate practices is important in the work of discussion leading. Following up on what students say is critical for establishing collective engagement. One way of following up on student responses is clarifying student thinking. Clarifying student thinking might be done, for example, to ensure that the rest of the class understood the contribution, to assess the contributing student’s understanding, or to determine the direction for the next steps in a discussion. One way to clarify student thinking is through revoicing, but other techniques could also be used.

The other product of this approach to decomposition is an articulation of *connections* among nested teaching practices. Moving along a strand from practices of larger grain size to those of smaller grain size details with increasing specificity how a practice can be implemented. In turn, starting with a technique and moving up a strand to practices of increasingly larger grain size makes visible the purpose that those techniques are serving. In this way, practices are both functionally and conceptually related. As shown in Figure 1, revoicing can be seen as an answer to successively asking, “How does a teacher do that?” at each step in the decomposition of leading a discussion. Asking “how” again would only yield components of revoicing, not another practice. In this way, a technique can be seen as the most fine-grained part of the work that has integrity as a teaching practice. In the other direction, revoicing can be seen as the starting point for repeatedly asking, “Why would a teacher do this?” The result is a string of questioning that ends with leading a discussion. The domain is the highest level of the work that still has integrity as a decomposition of teaching practice. Thus, decomposing teaching into nested practices of varying grain sizes simultaneously specifies and attends to the how and the why of practice.

Figure 1. Continuum of grain sizes of practice



Although it does aim to generate detailed articulations of practice, this approach to decomposition does not result in portrayals of teaching that are closed or confining. In this example, decomposing the work of leading a discussion led to revoicing by way of larger grain-sized practices such as clarifying student thinking and following up on student responses. Yet revoicing is not the only technique that could be used to accomplish these larger practices. For example, clarifying student thinking could also be accomplished by requesting a representation, asking a probing question, or having the student talk about a related problem. Similarly, there are other ways of nesting revoicing in the larger work of mathematics teaching. For example, revoicing can be seen as a technique in the service of the practices of facilitating connections (e.g., by making content available for students to link to previous ideas) or encouraging attention to the contributions of others (e.g., using it to indicate the value of what classmates say). Thus, revoicing is a high-leverage technique because it can service a number of different purposes.

In sum, our approach to decomposition is aimed at mapping teaching in ways that help beginning teachers simultaneously attend to the how and the why of practice. Beginning teachers can learn to enact particular practices and, at the same time, know they are addressing major components of teaching. This creates an architecture of practice can help them assimilate new knowledge and identify gaps within their own teaching practice. Beginning teachers can learn that teaching is interconnected work and that the same techniques can be used in different ways depending on one's instructional goals. This approach to decomposition also has benefits for teacher educators. It provides sense of the leverage of particular practices based on frequency and location in the decomposition. Teacher educators can thus focus less on naming every piece of teaching and more on developing nested high-leverage practices within each domain. The approach provides teacher educators with a flexible perspective toward the work of teaching that allows them to focus on techniques when beginning teachers need to work on enactment, and on domains when beginning teachers need a greater perspective on purposes and instructional goals.

We use this approach to decomposition in the design of our methods course. We focus on four domains of practice: leading a discussion; planning mathematics lessons; assessing students' knowledge, skill, and dispositions; and representing mathematical ideas. For each domain, we have articulated nested strands of practice, down to the technique level, that are the content of our course. We then use these decompositions to design the approximations of practice in which beginning teachers engage. In the next section, we use data from our course to illustrate the use of decompositions, representations, and approximations of practice to teach beginning teachers to lead discussions in mathematics.

TEACHING BEGINNING TEACHERS TO LEAD DISCUSSIONS IN MATHEMATICS

In this section, we illustrate our approach to helping beginning teachers learn to lead discussions. We begin with an overview of how we have decomposed the work of leading a mathematics discussion. We then present some of the representations and approximations of practice that we have used to engage beginning teachers in learning to do this work.

DECOMPOSING THE WORK OF LEADING A MATHEMATICS DISCUSSION

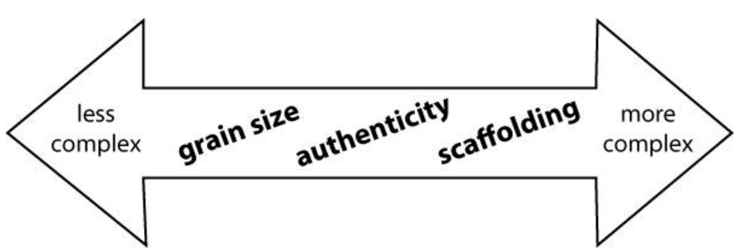
Decomposing any aspect of teaching practice is challenging, but decomposing the work of leading a discussion makes some of the challenges particularly salient. First, discussions are, by definition, interactive. Many of the teacher's moves are contingent on what students do as well as the ideas, methods, and questions they raise. Therefore, it is not possible to articulate exactly what a teacher will say or do in a discussion, or the order in which specific actions will occur. Second, the moves that are appropriate to make when leading a discussion depend on the content of the discussion and its instructional purpose. For example, whether and how a teacher takes up a particular student question or pursues a particular solution method depends on factors such as the mathematical point of the discussion (Sleep, 2009), when in the discussion the question or solution arises, and what else has already been discussed. And, finally, classrooms in which beginning teachers are working vary in the extent to which students engage in mathematics discussions. This means that norms and routines for productive participation in mathematics discussions may not have been established in some classrooms, which adds a layer of complexity to the work of leading the discussion and requires teaching moves that would not be necessary if students had more experience discussing mathematics.

Despite these challenges, we have been working to decompose what is involved in leading a mathematics discussion so that we can help beginning teachers learn to do this work. At the larger grain size, we identified five types of work associated with leading a discussion: two that set the stage for the discussion (setting up the task and monitoring student work), and three involved in the discussion itself (launching the discussion, orchestrating the discussion, and concluding the discussion). These categories of work resonate with other ways that discussions have been conceptualized in the field (e.g., Lampert, 2001; Smith et al., 2009). The categories provide an overarching structure for a discussion and are also generic and at a large enough grain size that they are applicable to the leading of many types of discussions in mathematics. For each of the five components, we have tried to further detail what teachers do when planning for and enacting that part of the discussion. This includes identifying what teachers are trying to accomplish, considerations that teachers make, and specific moves that can be used. Finally, we have tried to articulate what competent performance looks like in each category. In the next section, we further elaborate and provide examples of the ways in which we have decomposed the work of leading a mathematics discussion.

TRAJECTORY OF WORK ON LEADING DISCUSSIONS

Two central ideas have shaped our efforts to help beginning teachers learn to lead discussions, in particular, the approximations of practice in which beginning teachers have opportunities to engage in our course. The first idea is that, over the span of the semester, we want beginning teachers to have a series of opportunities to engage in increasingly complex approximations of practice. We view complexity as related to the authenticity of the practice, its grain size, and the amount of scaffolding provided by the teacher educator (Figure 2). We increase complexity over time by combining more authentic versions of larger grain sizes of practice with less scaffolding. Our aim of providing multiple opportunities to practice impacts the design of our course in a number of ways. For example, because we want beginning teachers to have repeated opportunities to practice in each of our focal domains, we layer and progressively integrate work on the domains throughout the entire course (as opposed to, say, working for 3 weeks on one domain, 3 weeks on the next, and so on).

Figure 2. Factors influencing the complexity of approximations of practice



A second central idea is that, because our goal is to teach beginning teachers to enact teaching, we also aim to assess the enactment of practice, not simply descriptions of or reflection on enacted practice. Quite often, methods courses focus assessment on extensive projects, curriculum units, elaborate written papers, or work that is exclusively in one-on-one settings with students. However, because practice is a goal of beginning teacher learning, providing feedback on the enactment of practice and gauging the skill with which beginning teachers engage in practice over time are essential. The aim of assessing enacted practice influences the design of our course and has implications for the ways in which we decompose practice and the types of representations of practice we both provide and require beginners to create. We map backward from our expectations to ensure that beginning teachers have opportunities to practice and get feedback on enacted practice before they are held accountable in summative assessments. We develop assessments with guidelines for creating multimedia records that provide sufficient evidence of that practice to support feedback and reflection (Moss et al., 2008). We also try to provide scaffolds that set up beginning teachers for successful enactment during assessments. For example, in early experiences, we provide feedback on their plans before they lead discussions on which they will be assessed.

To explore how these ideas play out in our course, we focus on three approximations of practice that occur at different time points: our initial work on leading a discussion, which focuses on asking purposeful questions; a midterm activity in which beginning teachers teach a "mini-problem"[3] in their field placement classroom; and a culminating assessment activity in which beginning teachers teach an entire mathematics lesson from their classroom's textbook. By focusing on approximations of practice at different points in the course, we seek to illustrate our efforts to develop a trajectory of repeated opportunities of increasing complexity and to assess the enactment of practice.

Asking purposeful questions. Questions are an important tool in teaching. The questions teachers ask impact students' opportunities to learn by shaping both the content with which students engage and the types of participation that are made available (Danielson, 1996; Martino & Maher, 1999). Yet asking questions is at the technique level. To try to move beyond technique, we focus not only on formulating individual questions but also on developing a framework of purposes that questions can serve. We use this connection to purposes to situate the technique of asking strategic questions in the larger domains of practice and to convey the big idea that teaching is deliberate and purposeful work.

We use a representation of teaching—a video of a whole-class discussion in a fifth-grade mathematics class—to begin our work on asking questions. Before showing the video, we introduce the notion of "teacher move" as something a teacher does or says during instruction and explain that we are studying teacher moves to help beginning teachers develop a repertoire of moves that they can deploy when they are teaching. We emphasize that teaching is deliberate: Teachers do not randomly pick moves; they purposefully choose moves based on the goals they are trying to achieve. A particular purpose can be attained by a number of different moves, and a certain move can serve a variety of purposes. Beginning teachers then watch the video and use a transcript to identify teacher moves and their possible purposes.

The video is not the only representation of teaching used to launch our study of teacher questions. In that same class session, beginning teachers participate in a "fishbowl" simulation activity[4] and, when they are in the role of observer, record all teacher questions. As part of that class's assignment, they read from *Classroom Discussions: Using Math Talk to Help Students Learn* (Chapin et al., 2003) and compare the "productive talk moves" presented in the text[5] with the moves identified in their analyses of the representations of practice studied in class. This assignment also provides a first opportunity to practice generating questions. Beginning teachers model given numbers with place-value materials and design questions they would ask students about the numbers.

Work on teacher moves and questions continues in the next class meeting. We begin with a discussion of the productive talk moves from the Chapin et al. (2003) book and the purposes for which a teacher might use the different moves in practice. However, we note that working in this direction is unrealistic: Teachers do not go into their classroom saying that they are going to revoice; instead, teachers have instructional purposes and enact moves that help them accomplish those purposes. We introduce a framework that articulates a range of purposes for teacher questions, and beginning teachers work in groups to generate actual questions that could be used to achieve each purpose (Table 1). Although we encourage beginning teachers to draw on specific examples, we have them pose generic questions that can be "filled in" for a given context, for example, with the content being discussed or a particular student's name.

Table 1. Framework for Teacher Questions

Purposes of teacher questions *(generated by teacher educators)*	**Example questions** *(generated by beginning teachers)*
1. Initial eliciting of students' thinking	• *Does anyone have a solution they would like to share?* • *How did you begin working on this problem?* • *Does someone have a different idea?* • *What have you found so far?* • *Did anyone approach the problem in a different way?*
1. Probing students' answers a. Trying to figure out what a student means or is thinking when you don't understand what he or she is saying b. Checking whether right answers are supported by correct understanding c. Probing wrong answers to understand student thinking	• *How do you know?* • *So is what you're saying ____?* • *When you say ____, do you mean ___?* • *Could you explain a little more about what you are thinking?* • *Why did you ____?* • *How did you get ____?* • *Could you use [materials] to show us how that works?*
1. Focusing students to listen and respond to others' ideas	• *What do other people think?* • *How does what ____ said go along with what you were thinking?* • *Who can explain this using ____'s idea?* • *Would someone be willing to add on to what ____ said?*
1. Supporting students to make connections (e.g., between a model and a mathematical idea or a specific notation)	• *How is ____'s method similar to (or different from) ____'s method?* • *How does [one representation] correspond to [another representation]?* • *Can you think of another problem that is similar to this one?* • *How does that match what you wrote on the board?*
1. Guiding students to reason mathematically (e.g., make conjectures, state definitions, generalize, prove)	• *Can you explain why this is true?* • *Does this method always work?* • *What do these solutions have in common?* • *Have we found all the possible answers?* • *How do you know it works in all cases?*
1. Extending students' current thinking and assessing how far they can be stretched	• *Can you think of another way to solve this problem?* • *What would happen if the numbers were changed to _____?* • *Can you use this same method to solve _____?*

These early experiences asking questions are very far from authentic practice. Although beginning teachers do practice articulating actual questions (as opposed to saying what they *would* ask), and they get immediate feedback that can be used to revise and improve their questions, the questions are asked in a university classroom, and they are not posed to or answered by students about mathematics that is actually being taught. One way we try to approximate asking questions in response to particular students is by watching short video clips of students solving mathematic problems (from the work of Carpenter, Fennema, Franke, Levi, & Empson, 1999) and having beginning teachers ask "next questions." But this is still far from authentic.

As the term progresses, beginning teachers have opportunities to practice asking questions in more complex approximations of practice—complex in terms of both the authenticity and the amount of scaffolding provided. One way the complexity increases is that the questions are asked in real time in response to actual students, first in a one-on-one interview and then when circulating among students during a whole-class lesson.

Circulating still involves interacting with individual students but is more complex because it requires moving quickly from student to student in the context of the larger lesson; thus, a teacher has to interact with individuals or small groups while still maintaining an eye on the whole class. Across these experiences, they collect audio recordings, anecdotal notes, and examples of student work as the basis for feedback and personal reflection on their use of questions. Beginning teachers also practice asking questions in whole-class settings and incorporate what they have been learning about teacher questions and their purposes into the leading of whole-class discussions. In our course, the first approximation of practice that involves interacting with the whole class is teaching a mini-problem, which we discuss next.

Teaching a mini-problem. Mini-problem is the name we created for a short mathematics problem or task used for review, extension, or warm-up. An appropriate mini-problem should be designed to enable pupils to get started and work on the problem independently, with minimal instruction from the teacher, yet it should engage each of the students productively. Though the name *mini-problem* is nonstandard, these types of short mathematics tasks are a common feature of elementary school classrooms. The mini-problem assignment is designed to provide beginning teachers with opportunities to practice skills in three of the focal domains of our course—planning, assessing, and leading a discussion. We focus here on leading a discussion.

As with asking questions, we begin work on the mini-problem with the analysis of a representation of teaching: a video from a fourth-grade class engaging in a warm-up problem. Before watching the video, beginning teachers think about the mathematics task in which the students in the video are engaging, as well as students' likely methods and misconceptions. We then introduce the Discussion Planning Framework (see the appendix). The framework decomposes the leading of a discussion into five stages: (1) setting up the problem; (2) monitoring student work; (3) launching the discussion; (4) orchestrating the discussion; and (5) concluding the discussion. It also decomposes the work of planning for discussions by listing considerations that teachers make to plan for each of these stages. Beginning teachers watch the video, which has been partitioned into clips to match the stages in the framework. As they watch the video, they identify specific moves that the teacher makes in that phase of the discussion and generate other moves that might have been made instead. These examples are intended to help illustrate and further unpack the Discussion Planning Framework.

Beginning teachers use the Discussion Planning Framework to plan the mini-problems they will be teaching in the field. Before teaching, they submit their plan to their course instructor and get detailed feedback. They then teach the problem in the field, making a digital recording and collecting any artifacts (e.g., student work) that are produced. They use the recording to analyze their questioning techniques. In their analysis, they identify five key questions that were used during the segment. Several of the questions need to show the use of questioning techniques discussed in class. They describe the thinking behind their use of each question, which might include comments about the instructional purposes, the phrasing used, to whom the question was posed, and so on. They summarize how students responded and describe how this response relates to the purposes of the question. They then select three of the five questions to improve, perhaps by rewording the questions or designing new questions that better serve their purposes at that point in the discussion.

Beginning teachers also select a segment from their discussion to share with small groups of their peers. These conversations focus on describing what happened during the mini-problem, whether the problem provided students with opportunities to reason about important mathematical ideas and discuss different solution methods, and the moves and challenges they faced in supporting students' engagement and keeping it focused on important aspects of the mathematics. Based on their discussions, we revise the Discussion Planning Framework, and they identify an aspect of discussion leading that they want to work on in their future discussions.

The mini-problem is an approximation of practice that is more complex than the initial work on asking questions. Beginning teachers have to manage a whole class of students and engage and respond to them in real time, which positions the mini-problem relatively high on the authenticity scale. However, the complexity is reduced by the nature of the task that the teacher is using—a short, self-contained problem. We also support beginning teachers in selecting a problem by providing some examples of appropriate problems that work well in this type of short discussion. In addition, because the problem is supposed to be something that students can begin work on independently, beginning teachers need not worry about setting up the problem and can therefore focus on the discussion portion. The complexity is also reduced through the significant amount of scaffolding that is provided, for example, through the written framework and question shells, as well as through the specific, detailed feedback received before teaching. This feedback helps beginning teachers avoid the pitfalls and mathematical errors that are visible in their plan and suggests additional considerations to help them better prepare for the discussion.

Teaching a whole-class mathematics lesson. The third approximation of practice we discuss is one of the "culminating performance assessments" for the course. We have four culminating performance assessments, one for each of the focal domains. Three of the assessments are based on a whole-class mathematics lesson that is taught in the field. The three lesson-based culminating performance assessments draw on the skills across the domains of planning, leading a discussion, and assessing, and provide an opportunity to integrate them in an authentic approximation of practice.

Beginning teachers use the various lesson analysis and planning tools provided in the course to plan their lesson. Before teaching their lesson, they have a "lesson conference" with their course instructor. This conference serves two purposes: to evaluate their planning performance (the lesson conference serves as the culminating performance assessment for the planning domain), and to get feedback that they can use to improve their lesson. In particular, instructors try to provide feedback that sets them up to have a productive mathematics discussion, because the discussion in this lesson serves as the culminating performance for the leading a discussion domain. Instructors consider and give feedback on the "discussability" of the task, the goals for the discussion, the proposed launching and concluding moves, and the anticipated student responses.

Beginning teachers are told that the enactment of their discussion will be evaluated (using a video or audio recording) based on the questions listed in Table 2. In addition to submitting the recording, beginning teachers write a one- to two-paragraph explanation of how the discussion reflects their ability to skillfully perform one of the foci listed in Table 2. In their write up, they are asked to cite specific examples from their discussion. Instructors use a grading tool to score and provide feedback on the enacted discussion.

Table 2. Teaching a Mathematics Lesson: Questions for Analysis and Evaluation of Discussion

Questions guiding the teacher educator's evaluation of the enactment:

- Does the mathematics problem allow for discussion?
- Does the teacher launch the discussion to elicit initial contributions?
- Is the discussion focused on mathematics?
- Does the teacher solicit broad participation?
- Does the teacher use a variety of moves that:
    - probe students' contributions?
    - connect students' ideas?
    - encourage students to consider and respond to classmates' ideas?
    - guide students to reason mathematically?
    - extend students' thinking?
- Does the teacher conclude the discussion?

Possible foci for beginning teachers' analysis of their enactment:

- Purposefully using questions to elicit, probe, and connect students' mathematical ideas
- Supporting students to consider and respond to their classmates' mathematical ideas
- Supporting students' reasoning about or explanation of a particular mathematical idea
- Helping students make explicit connections between representations and/or solution strategies
- Concluding the discussion in a way that highlights the main mathematical content of the lesson and goals of the discussion
- Fielding a student response that you anticipated or did not anticipate in your planning
- Attending to and engaging all students' participation in the discussion
- Mediating the context of the lesson to attend to issues of equity
- Improving the practice you identified as something you wanted to work on from your mini-problem discussion

Teaching a whole-class lesson is the most complex approximation of practice in our course. Beginning teachers need to identify parts of their lesson that are appropriate for whole-class discussion and need to transition in and out of these parts, which may or may not materialize as anticipated during the lesson's enactment. Although it is high on the authenticity scale, it still includes artificial aspects. For example, in practice, teachers do not focus on individual lessons in isolation, but rather plan lessons as part of a sequence of instruction. There is also a substantial amount of scaffolding provided, both in the planning tools and in the feedback that is given before the lesson is enacted.

ANALYSIS OF OUR TRAJECTORY OF WORK ON LEADING DISCUSSIONS

In the preceding descriptions, we tried to illustrate our efforts to engage beginning teachers in recursive cycles of increasingly complex practice that address both the how and why of practice by connecting specific techniques to larger domains of practice. We also traced the progression of documentation and explicitness of facets of enacted practice that are assessed. In general, we use representations of teaching to introduce an aspect of practice and to unpack key features of the work. Beginning teachers' engagement in the practice is then scaffolded by frameworks and other written decompositions of practice, and they have opportunities to rehearse in class and/or get feedback before trying out the practice in the field. They collect records of their field-based enactment, which they use to reflect on their practice, to get feedback from their instructor and colleagues, and to submit for summative assessment.

Analysis of our work reveals additional design considerations, insights, and questions about pedagogies of practice in professional education. One observation, perhaps obvious, is that we use different types of representations of practice for different purposes throughout our course. For example, to introduce a new practice, we often use a more composed representation of practice that we know illustrates the focal aspects of practice, such as a 5- to 10-minute video clip from a whole-class mathematics lesson. Using a more composed representation enables the practice we are aiming to study to be situated in the larger context of teaching work. But using composed representations to analyze a specific practice can also be challenging because there is so much else that can be noticed in addition to the focal practice. To make such representations of practice "studyable" (Ghousseini & Sleep, 2011), we scaffold beginning teachers' viewing, and to focus their attention, we provide viewing lenses. These lenses take a variety of forms. For example, a lens might be a simple focus question (e.g., asking beginning teachers to identify teacher moves); in other cases, the lenses are more complex, such as the Discussion Planning Framework. Another important type of representation used in our course consists of those created by beginning teachers as they engage in approximations of practice. These representations take a variety of forms and enable beginning teachers to reflect and receive feedback on the enactment of their discussions. Representations were often composed of bundles of records of practice, some that captured the enactment of a teaching practice, and others that provided important context for sharing the practice with those outside the setting of practice (such as instructors) or holding important information necessary for reflection. Components of these bundles could be drawn from documentation that emerges from typical classroom teaching (e.g., work samples, anecdotal notes), but others required extra steps to capture (e.g., audio, video) or were made for the sole purpose of the bundle (e.g., written reflection, question analysis). Focusing feedback and assessment on enactment raises key considerations for the design of approximations of practice, including the interaction of content, which we discuss in the next section.

Through our efforts to become more explicit about what beginning teachers are learning to do in the course (both for ourselves and for them) and to scaffold their engagement in practice, we have developed a number of written decompositions of practice. In many cases, these written decompositions specify options that the beginning teacher could say or do in a particular teaching context. Such specifications are themselves representations of teaching. Some of these written decompositions, such as the Discussion Planning Framework (see the appendix), represent a self-contained, discrete instantiation of teaching practice. This type of decomposition/representation seems to resonate with Grossman et al.'s (2009) notion of decomposition "in which the parts maintain an integrity of their own even as they invoke elements of the whole" (p. 2070). But not all our written decompositions reflect self-contained aspects of practice. For example, asking questions does not occur at a particular time in a lesson; teachers ask questions in a range of contexts for a range of purposes—for example, as part of whole-class discussions, when working with individual students, and when introducing new tasks. Instead of decomposing the enactment of a discrete practice, the question shells and their purposes serve as more of a "toolbox" from which beginning teachers can draw during their instruction. It is not clear whether or how such "cross-cutting practices" (TEI, 2009) fit into the Grossman et al. framework.[6]

One advantage of working on cross-cutting techniques is that they can be practiced in a wide variety of teacher education activities. Naming the specific practices enables teacher educators to make visible the repeated opportunities to practice the techniques and to make connections to the work of teaching across a range of class work. For example, we were able to position the student thinking interview assignment as another opportunity to practice asking questions, a skill they also used in leading whole-class discussions. Being able to name the practices that were being developed in different assignments and activities helped us distinguish the teacher education activity from the teaching practices that beginning teachers were learning.

As Grossman et al. (2009) noted, decomposition of practice "may require a specific technical language for describing the implicit grammar and for naming the parts" (p. 2069). However, beyond domains and a collection of techniques, teaching does not yet have an agreed-on technical vocabulary. Thus, in many cases, we had to invent our own terminology to try to capture other facets of the work. This could make it difficult for beginning teachers to connect what they are learning in our course to their other teacher-education experiences and to the representations of practice encountered in the field.

Another concern with naming small aspects of practice is that it can misrepresent teaching as a compilation of techniques. As discussed earlier, one way to address this issue is to maintain the connection between specific techniques and the domains they serve. In the design of approximations of practice in our course, we try to represent this connection by nesting the smaller cross-cutting techniques in larger discrete practices. In the case of leading a discussion, we nested asking questions in the discussion framework, which we then nested in the lesson. We think that such nesting of practices can be a useful way to recompose practice. Yet as practices are nested into more complex practices, it becomes increasingly difficult to scaffold beginning teachers' enactment through specification. This is another reason that we have found working on cross-cutting techniques to be useful; beginning teachers can draw from their toolbox of teacher moves to customize the more general frameworks for their specific context.

IMPLICATIONS FOR CONCEPTUALIZING PEDAGOGIES OF PRACTICE IN TEACHER EDUCATION

We have described our approach to decomposing practice and the ways in which that approach serves as a foundation for the use of representations of teaching and the design and sequencing of approximations of practice. Analyzing our efforts has raised some issues that seem critical to the use of pedagogies of practice in teacher education. We close by considering two issues that are relevant for the ongoing conceptualization pedagogies of practice: the interaction of assessment and of disciplinary content with decomposition, approximation, and representation of teaching.

DECOMPOSITION, APPROXIMATION, AND REPRESENTATION IN ASSESSING TEACHING PRACTICE

Assessment is needed for multiple purposes in teacher education, for example, to provide feedback to beginning teachers about their engagement practice, to support beginning teachers' self-assessment, to track progress of individuals and groups, to make consequential decisions about beginning teachers' readiness to teach, to make instructional decisions, to evaluate and improve the teacher education program, and to communicate with outside stakeholders about the preparedness of its graduates (Moss et al., 2008). The development of practice-based assessments is fundamentally tied to ideas of decomposition, approximation, and representation: Assessments must be grounded in articulations of the teaching practices that are generated through decomposition. Assessment activities necessary to support the learning of practice over time should capture engagement in increasingly complex approximations of practice. Records of practice that capture beginning teachers' work with pupils represent the beginner's teaching to those who provide feedback on and appraise practice.

Assessing beginning teachers' teaching has a number of implications for designing and implementing pedagogies of practice in teacher education. Practice needs to be parsed in ways that enable the decompositions to be used to develop assessments. For example, the same component of practice may even need to be described in different ways depending on the assessment context. This requires flexible language for describing practice, as well as depth. Not only do components of practice need to be clearly and perhaps differently named, but assessment places an additional burden of describing degrees of sophistication with the practice, perhaps with different articulations to meet the needs of particular users.

Approximations of practice also take on new dimensions when considered through the lens of assessment. To the extent possible, we aim to use enacted teaching practice as evidence of beginning teachers' learning (Boerst & Sleep, 2007). In our course, assessments occur in in-class simulations, brief and tightly framed engagement in classrooms, and extended experiences that integrate multiple domains of practice. The contexts in which the approximations of practice occur influence the enactment of practice and therefore have consequences for assessment (Moss, 2010). For example, suppose one wanted to appraise beginning teachers' ability to establish norms for a mathematics discussion. Whether this practice even occurs in a particular discussion hinges to some extent on the contextual demands. In some contexts, the discussion may require no explicit moves by the beginning teacher to establish norms (e.g., students already know how to participate in discussions), whereas in other cases, it may occupy a large proportion of teacher moves in the discussion. Thus, when designing and using approximations of practice for assessment purposes, it is important to manage contexts in such a way that beginning teachers have opportunities to show their skill and for teacher educators to account for differences in context when providing feedback.

Assessment considerations also come into play when sequencing approximations. One consideration in sequencing is determining the aspects of practice on which beginning teachers need feedback early or repeatedly. For example, early approximations need to provide opportunities for feedback on difficult or foundational aspects of practice. Subsequent approximations might then be selected on the potential they afford for beginning teachers to capitalize on previous feedback.

Assessment also influences the ways in which practice is represented. In many cases, beginning teachers are charged with representing their own practice for purposes of assessment or self-reflection; however, the ease with which different aspects of practice can be represented varies (Boerst, Sleep, Cole, & Ball, 2008). Some practices produce a "residue," like written student responses to a prompt, and thus representations are generated during enactment. Other aspects of practice can be captured with minimal additional preparation (e.g., positioning a stationary camera to capture a discussion), whereas others are not directly observable (e.g., a teacher's assessment of what students say during a discussion) and thus require additional effort to represent them. Teacher educators therefore need to identify, and then work with beginning teachers to document, those facets of practice that are essential for particular assessment purposes.

Representing practice for purposes of assessment requires more than documenting enacted practice. Those assessing practice also need to know about the particular context, for example, about the students, the curriculum, features of the school, and key goals or orientations of the teacher (DIAS Project, 2009). At the same time, it is important not to overwhelm or unnecessarily burden the beginning teacher with time-consuming requests for contextual information. Further research is needed to learn more about which types, or what degree, of contextual information are necessary for particular assessment purposes.

THE INTERACTION BETWEEN DISCIPLINARY CONTENT AND PEDAGOGIES OF PRACTICE

Another key consideration in the design and use of pedagogies of practice in teacher education is the specific subject matter content—in this case, mathematics. In our work on leading discussions, we found that the mathematical content of the discussion affected beginning teachers' practice in a number of significant ways. First, the complexity of the mathematics has consequences for the difficulty of managing a discussion. For example, some mathematical ideas are more difficult for students (and teachers) to understand; other topics might be particularly language intensive or have complicated representations to explain or deploy. Some mathematics content is also more "discussable," for example, topics that lend themselves to open-ended problems or have multiple representations. When beginning teachers work on a lesson in which the mathematics problem is not well-suited for a class discussion, their opportunity to practice setting up and leading a discussion is limited. The complexity and discussability of the content have implications for assessment, and teacher educators need to take the specific content into consideration as one of many contextual factors.

In addition to the complexity of the content and the discussability of the task or problem, mathematics discussions also entail the use of mathematical practices. Attention to definitions, for example, is often important. Disagreements in class may be a product of unexamined multiple definitions rather than about interpretations or solutions. Teachers need to be able to distinguish these and also to have ways to call attention to and develop common definitions. Another mathematical practice central to discussions is explanation. Teachers' own understanding of the specific features of mathematical explanation matters for both what they say and how they say it, as well as how they guide pupils' efforts to explain. Still another mathematical practice central to the practice of leading discussions is representing mathematical ideas and procedures. Teachers represent ideas in the course of discussions, and they also support and respond to pupils' representations. Again, their awareness of key aspects of mathematical representation is important in ways that go beyond general considerations of instructional representation. In fact, a generic view of the role of representation in a math discussion could lead unwittingly to a distortion of the content. For example, thinking that representations are important for capturing pupils' interest and engagement may lead a teacher to use a "fun" model (such a story about missing pies to teach addition of integers) that misrepresents the meaning of the content. The synergy between mathematical practices and the pedagogical practices of leading a mathematical discussion offers resources for enriching beginning teachers' capacity to lead substantive discussions. Developing ways to maximize these connections and guide beginning teachers in learning to lead discussions of high mathematical quality is an area of our current ongoing work.

Because content is at the heart of teaching, it is crucial to attend to the ways in which it interacts with pedagogies of practice—both when teaching beginning teachers to do the work and when assessing their progress. Specifically, decompositions need to embody the centrality of content in teaching, and the nature of sophistication and quality of teaching practice must convey mathematical considerations. For example, decompositions may include intermediate- and technique-level practices that are unique to the mathematics being taught, and decompositions may differ across different strands of mathematics. Approximations can be made more or less complicated by manipulating the selection of content. Focusing on particular content can scaffold beginning teachers; however, specifying the content in an approximation of practice may complicate enactment when there is a mismatch with the mathematics currently being studied in the field. Representations of practice must capture facets that are central to mathematics teaching. For instance, because representations are important in mathematics, beginning teachers might be required to use video to capture their practice, whereas an audio recording would suffice in other content areas. Understanding representations of practice may require the documentation of contextual information that is unique to mathematics instruction (e.g., curriculum, tools). Additional research is needed to explore the interaction of content with the pedagogies of practice and to determine productive ways to address content-related issues that arise.

CONCLUSION

In this article, we used Grossman and colleagues' (2009) framework for pedagogies of practice in professional education to describe our efforts to develop beginning teachers' skills in leading a whole-class discussion in mathematics. Our analysis identified a number of issues that may be relevant for the ongoing conceptualization of pedagogies of practice and may improve our capacity to teach practice. First, our work suggests that decomposing teaching into nested practices of varying grain sizes that maintain the connection between techniques and domains can support beginning teachers in attending simultaneously to the how and the why of practice, as well as provide a map of teaching that teacher educators can use flexibly in supporting the learning of teaching. Second, nesting early approximations inside subsequent ones is a way to support beginning teachers in building toward recomposed teaching practice. As practices are nested, teacher educators can increase the complexity of the approximations by adjusting the authenticity of the context of practice, the scaffolding of facets of the approximation, and perhaps the content central to the teaching. Third, we argue that assessment is vital to pedagogies of practice. Decompositions, approximations, and representations of practice therefore need to be developed in ways that support assessment for the array of teacher education purposes. And, finally, because content is at the heart of teaching, it is crucial to attend to the ways in which content interacts with pedagogies of practice. We believe these points simultaneously attest to the utility of Grossman's framework and to the promise of further work in these areas.

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expressed in this paper do not necessarily reflect the position, policy, or endorsement of the foundation.

Notes

1. The authors have all been core participants in this group, some of us since its inception in 1997.
2. We wanted to create a full collection of records from the course so that they could be studied and used by future instructors learning to enact specific aspects of the course.
3. The term *mini-problem* is our unconventional name for a short mathematics problem, often called a warm-up problem, sponge, or boardwork. We had originally named this activity *teaching a warm-up problem* but ran into problems because this name incorrectly conveyed that the problem was supposed to be done at the beginning of math time and had to relate to the topic of the lesson. We settled on *mini-problem* to convey the shortness of the problem; however, there are a number of problems with this term that we discuss later in the article.
4. In this fishbowl activity, half of the group works on the floor with the instructor, engaging in a primary activity at an adult pace; the other half observes, recording all the questions that are being asked by the "teacher."
5. Chapin et al. (2003) focused on the following teacher moves: revoicing; asking students to restate someone else's reasoning; asking students to apply their own reasoning to someone else's reasoning; prompting students for further participation; and using wait time. They use transcripts and other written representations of teaching to define these moves and illustrate their use in practice.
6. Cross-cutting practices are not synonymous with techniques. Technique reflects the grain size of practice: It is a specific teacher move (i.e., small grain size). Cross-cutting reflects the fact that it occurs across a range of teaching contexts and formats. A cross-cutting practice could be of a small grain size (e.g., asking questions) or of a large grain size (e.g., differentiating instruction).

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APPENDIX

Discussion Planning Framework

Analyzing the problem:

1. Identifying the mathematical content and instructional purpose
   - Do the problem yourself.
   - What is the mathematics that students are supposed to be working on?
   - What is your instructional purpose for using the problem?

2. Anticipating student thinking
   - What knowledge or skills will students need in order to do the problem?
   - What methods are students likely to use? What solutions or responses are students likely to generate?
   - What misconceptions are students likely to have? What errors do you anticipate they will make?

Planning its enactment:

1. Setting up the problem
   - How will you present the problem to students?
   - What materials will you and the students need?
   - If needed, how will you familiarize students with representations used in the problem?

1. Monitoring student work
   - What would you look for or listen for to determine if students are engaging with the problem?
   - What will you do to learn about and keep track of what students do as they engage with the problem (e.g., what strategies students are using, what errors they are making) that might inform how you lead the discussion?

1. Launching the discussion
   - How will you start the discussion of the problem solutions and strategies? (i.e., what specific question, example, or idea will you use?)

1. Orchestrating the discussion
   - *Student strategies:* What are some prompts you will use to elicit, probe, and connect students' solutions and strategies?[1]
   - *Focal mathematical ideas:* What might you ask to support students' reasoning about the important mathematical ideas in the problem? *(You should include examples of how you will respond to anticipated student errors or incomplete solutions.)*
   - *Interactions:* What will you say or do to encourage students to consider the thinking of others and respond to each other in the discussion?

1. Concluding the discussion
   - What specific statement will you use to begin wrapping up the discussion? *(It is possible that you will need to close the discussion before a natural place to stop presents itself, so consider how you will make this shift near the end of your time.)*
   - What mathematical ideas do you want to summarize at the end of the discussion?

[1] Your questions should include some of the discussion moves we have been learning about in class.

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